Reachability Analysis to perform robotic missions safely

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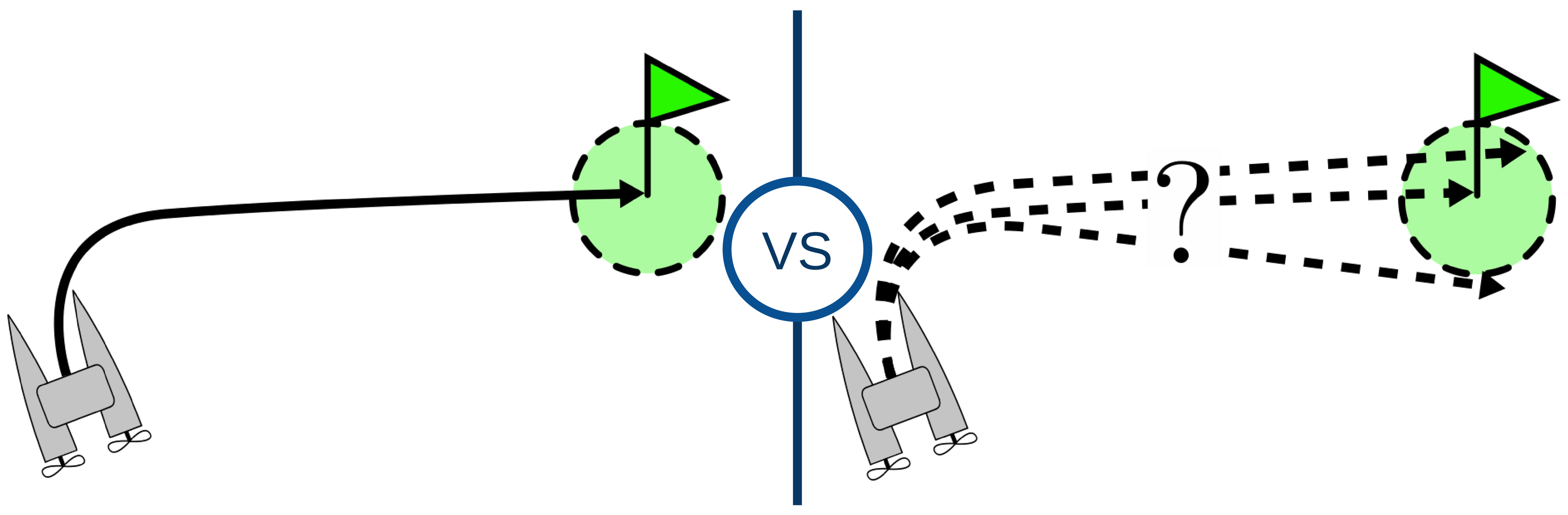
Problematic: How not to loose a robot ?

Before starting a mission with an autonomous robot it is important to make sure that it will **never be lost** or **damaged.**

As a robot behaviour can not be known exactly due to all the external disturbances that may perturb it, we need to compute all its possible trajectories.



Perfect case

Perturbated case

Reachable Sets are used to encapsulate all these possible states of the robot. They can be computed by performing a **Reachability Analysis.**

Reachable Set

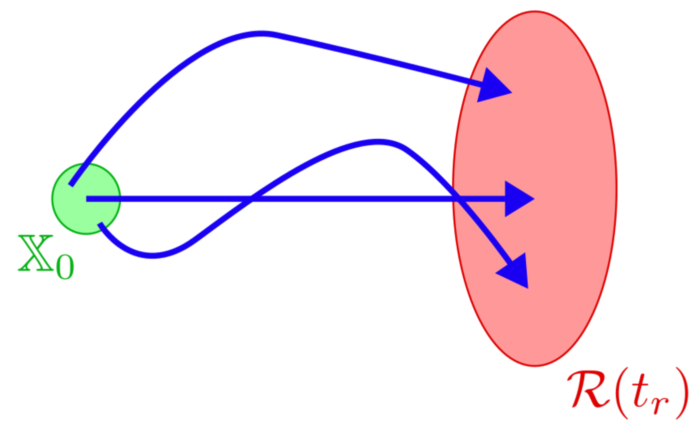
A robot is a dynamical system that follows a state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ where $\mathbf{x}$ is the robot state (*position, orientation …*) and $\mathbf{u}$ represents all the disturbances : the inputs (*motors, rudder …*) and the perturbations (*wind, current …*)

The robot initial state and the disturbances are bounded, they respectively belong to the sets $\mathbb{X}_0$ and $\mathbb{U}$.

The Reachable Set at a time t_r contains all the possible state of the robot at this time. It is described by :

$$\mathcal{R}(t_r) = \{\mathbf{y} | \mathbf{y} = \int_0^{t_r} \mathbf{f}\big(\mathbf{x}(t), \mathbf{u}(t)\big) dt\}$$

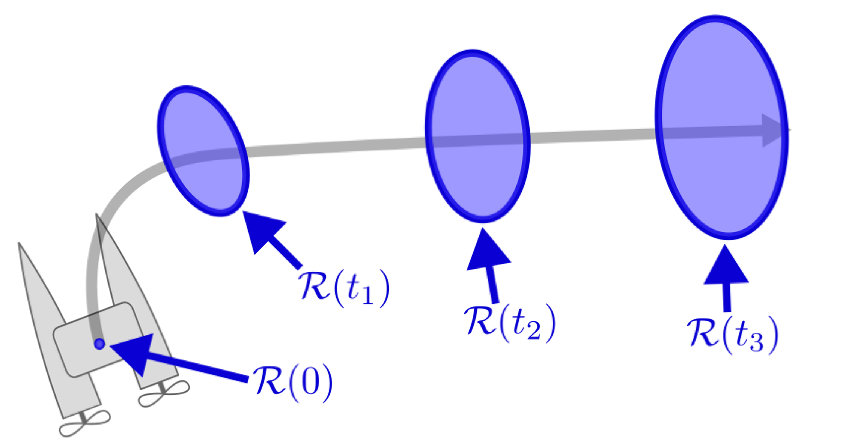
With $\mathbf{x}(0) \in \mathbb{X}_0$ and $\mathbf{u}[0, t_r] \in \mathbb{U}$



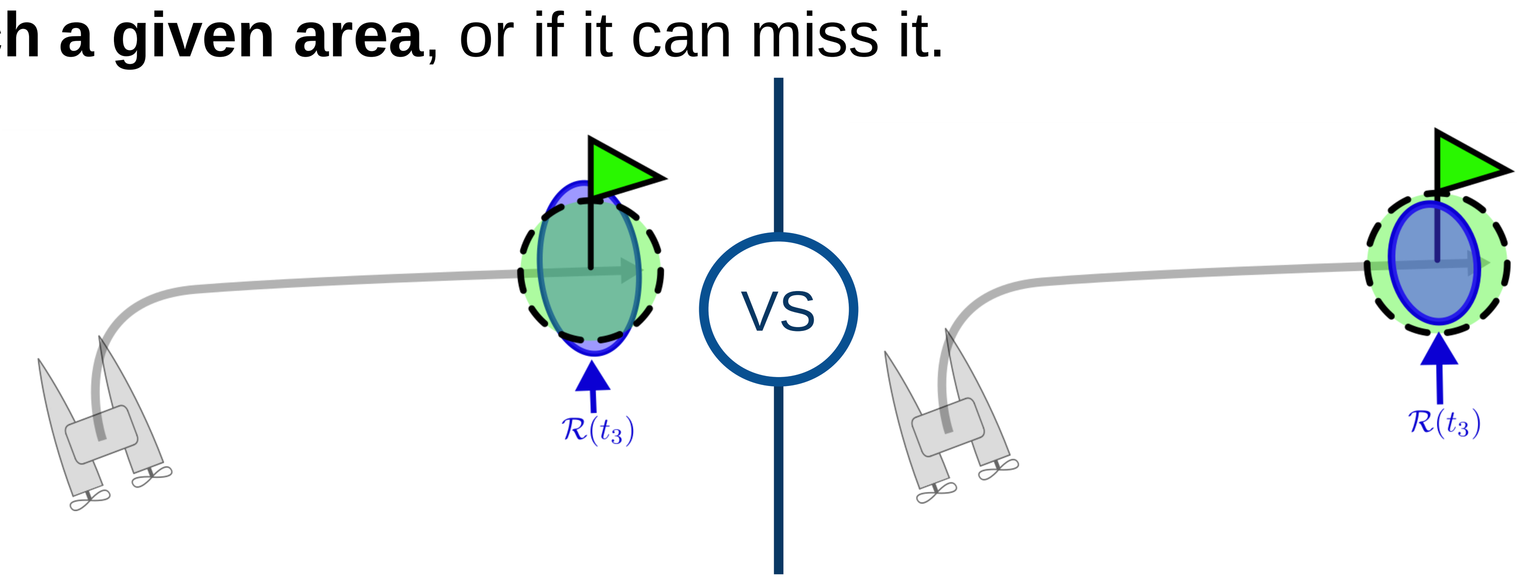
Representations and Use Cases

Discrete Representation

The Reachable Set can be represented in a **discrete** way. In this case, it is possible to say that at the distinct times t_1, t_2 … the robot will be somewhere inside $\mathcal{R}(t_1)$, $\mathcal{R}(t_2)$ …



This representation can be used to assert if a robot will **reach a given area**, or if it can miss it.

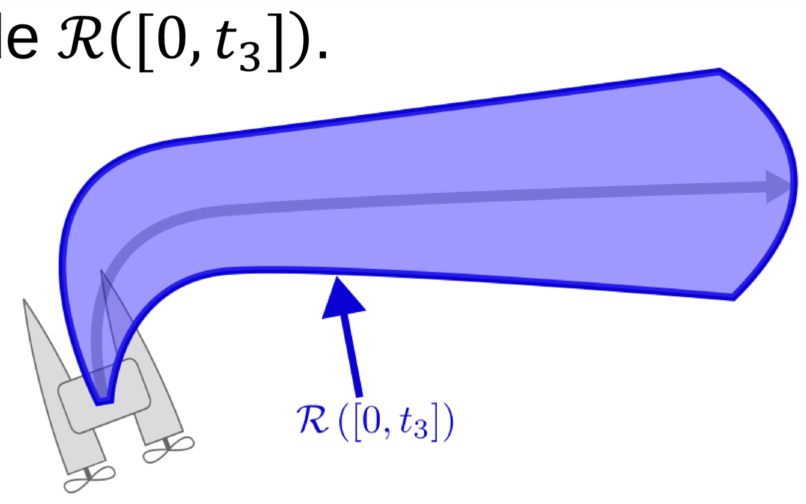


Unsafe case

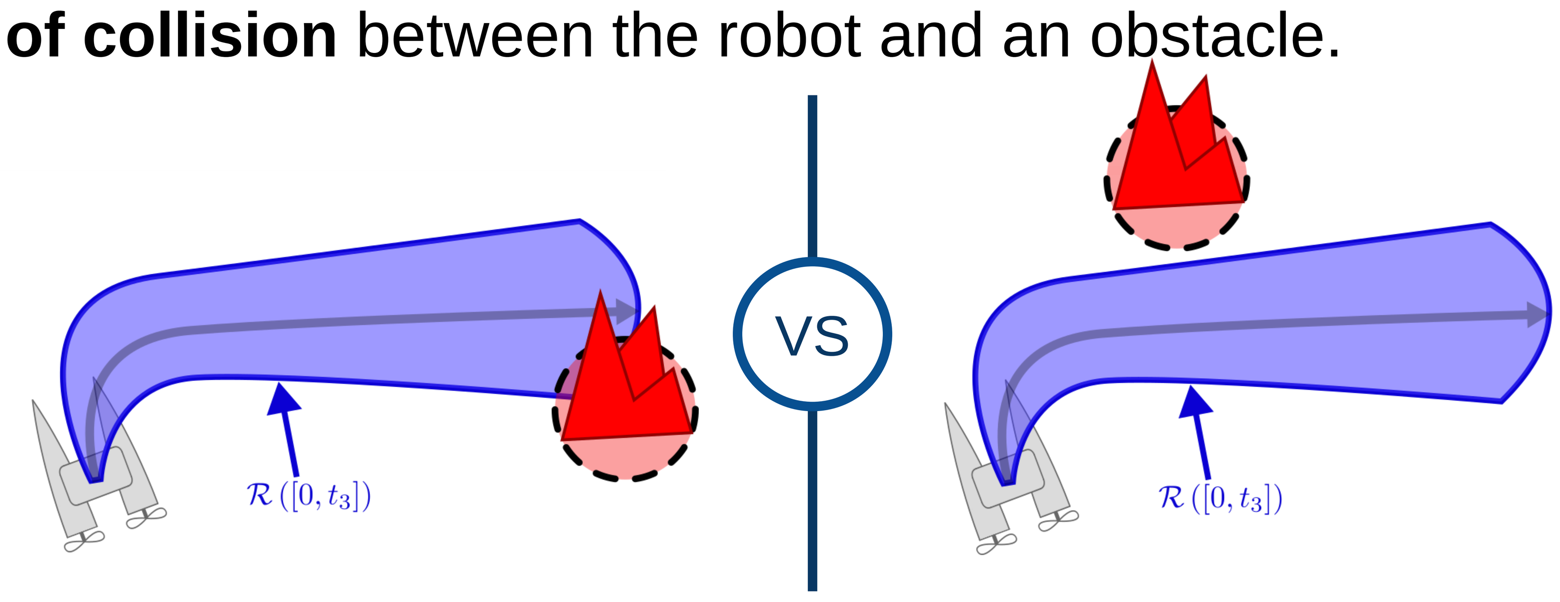
Safe case

Continuous Representation

The Reachable Set can be represented in a **continuous** way. In this case, it is possible to say that between the beginning of the mission and the time t_3 the robot will never be outside $\mathcal{R}([0, t_3])$.



This representation can be used to assert if there is a **risk of collision** between the robot and an obstacle.



Unsafe case

Safe case