Reachability Analysis to perform robotic missions safely

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Avec le soutien de

Summary

1. Introduction
2. Reachability Analysis
3. Use cases
4. Conclusion

Introductive Problem

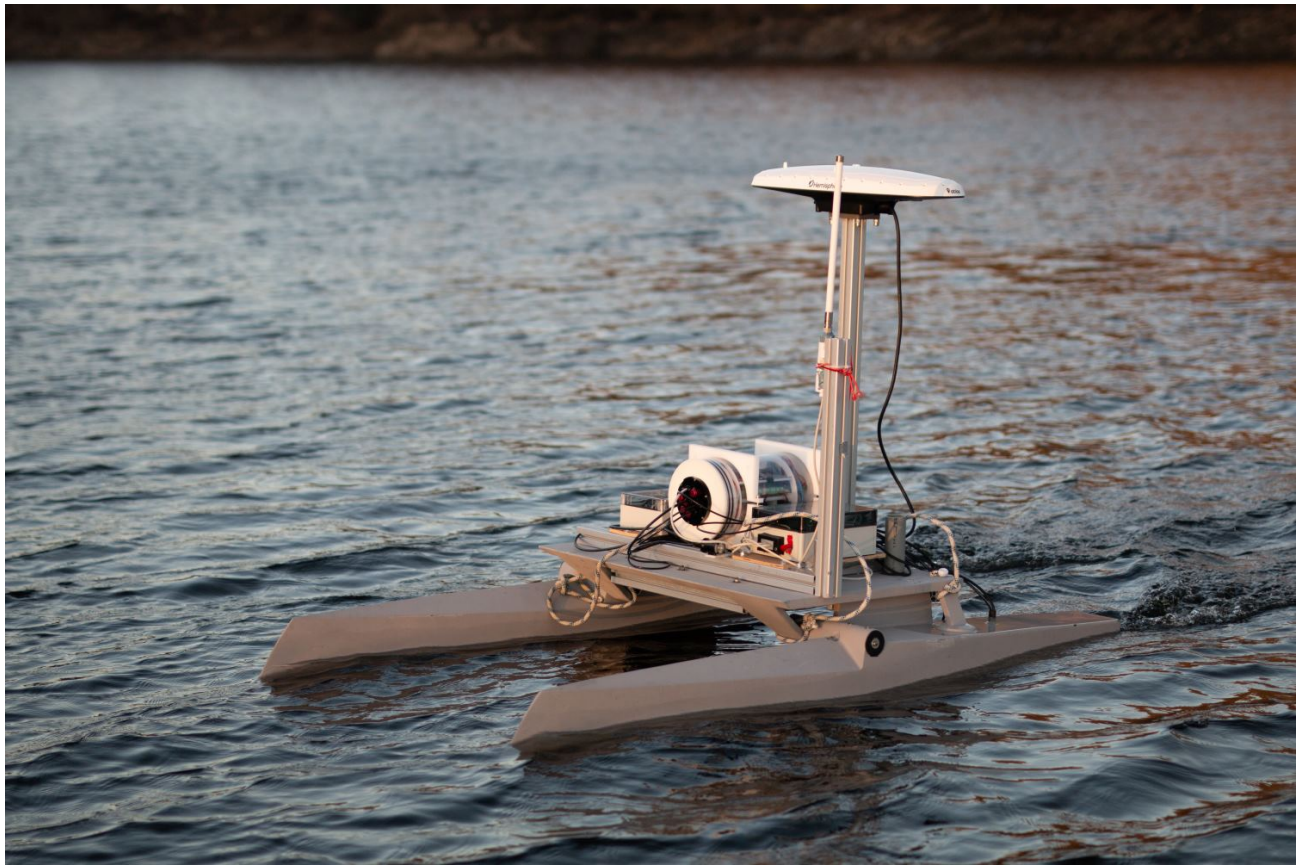

Figure: Helios

Introductive Problem

Figure: Base Situation

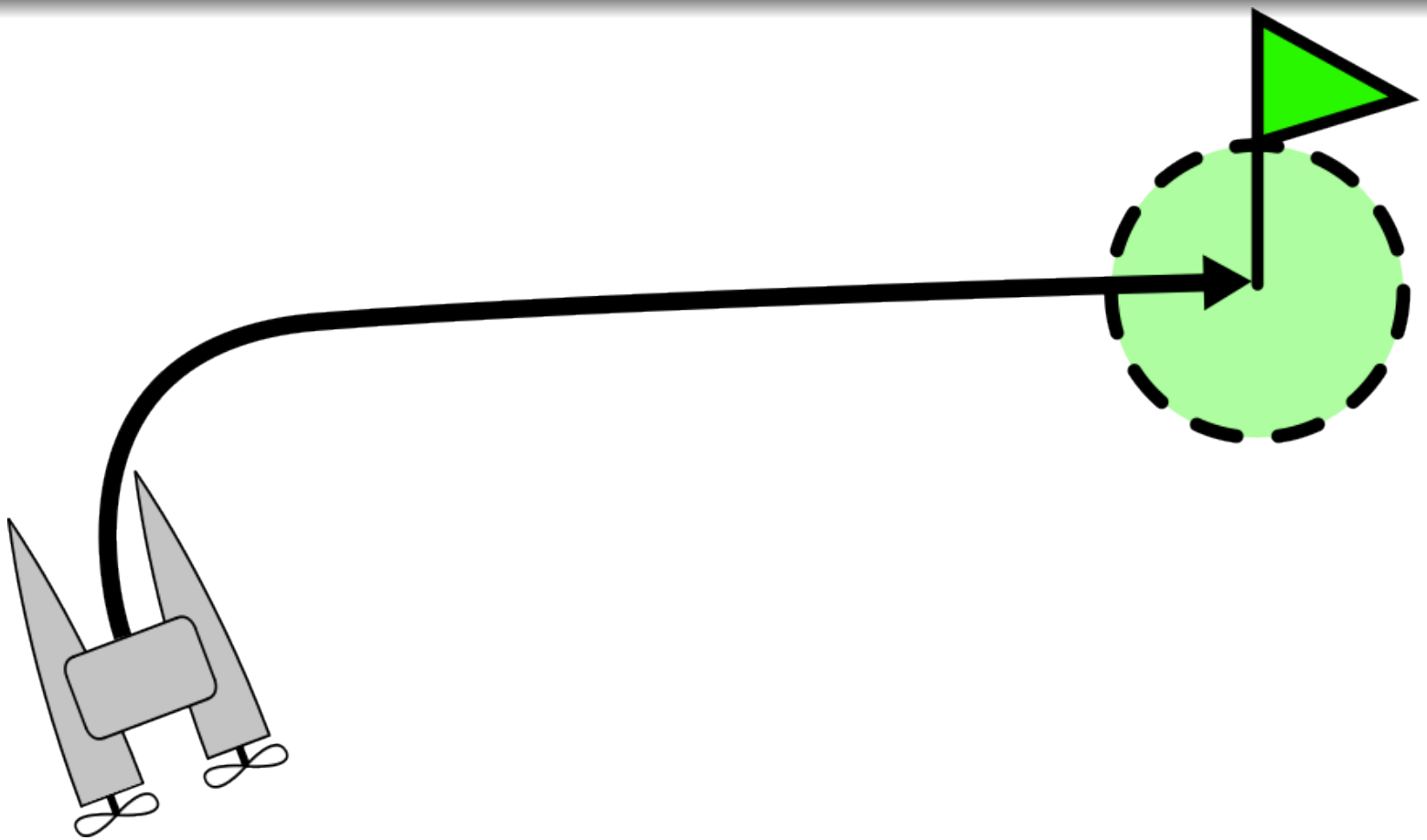

Figure: Perfect Case

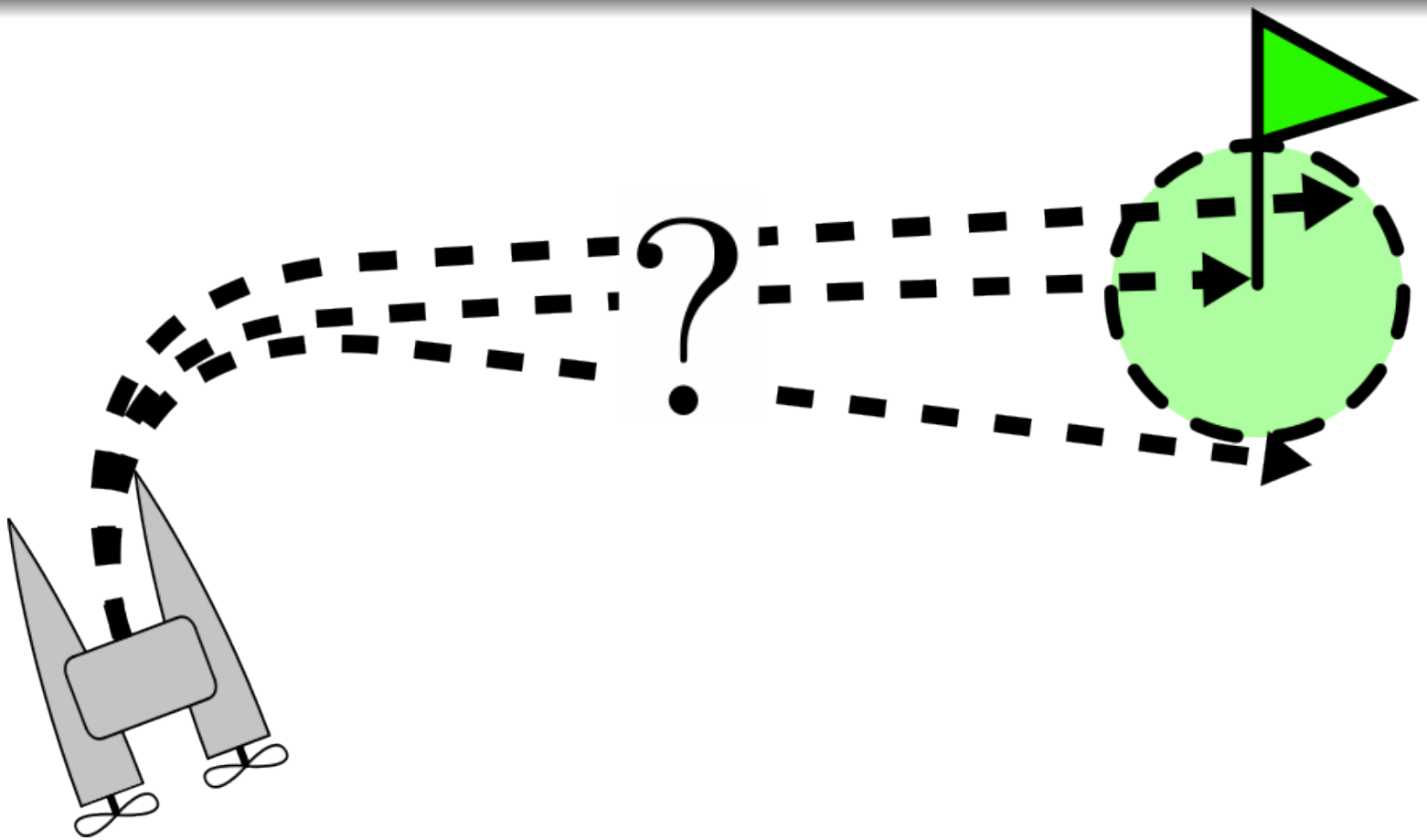

Figure: Perturbated Case

How can I compute the possible states of my robot ?

Definition

Robot state : x

State Equation : $\dot{x} = f(x, u)$

$u \in \mathbb{U}$: disturbance : inputs (motors) and perturbations (wind)

Definition (Reachable set at a point in time)

The Reachable Set at a given time t_r noted $\mathscr{R}(t_r)$ is defined by :

$$\mathscr{R}(t_r) = \left\{ y | y = \int_0^{t_r} f(x(t), u(t)) dt \right\}$$

With $x(0) \in \mathbb{X}_0$ the set of initial state and $u[0, t_r] \in \mathbb{U}$ the disturbance set

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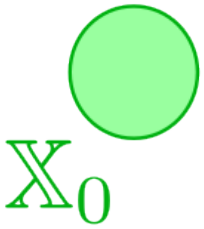
$\mathbb{X}_0$


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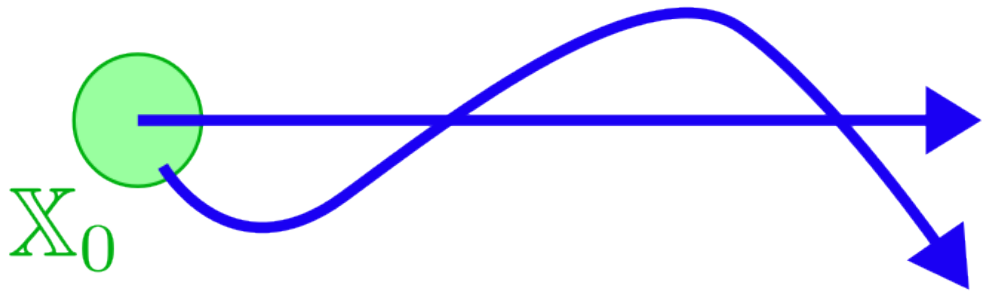
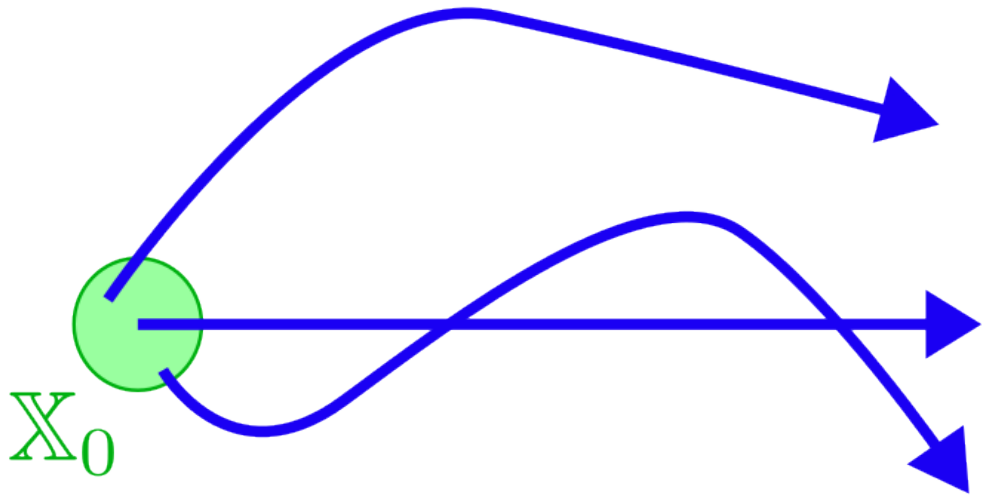
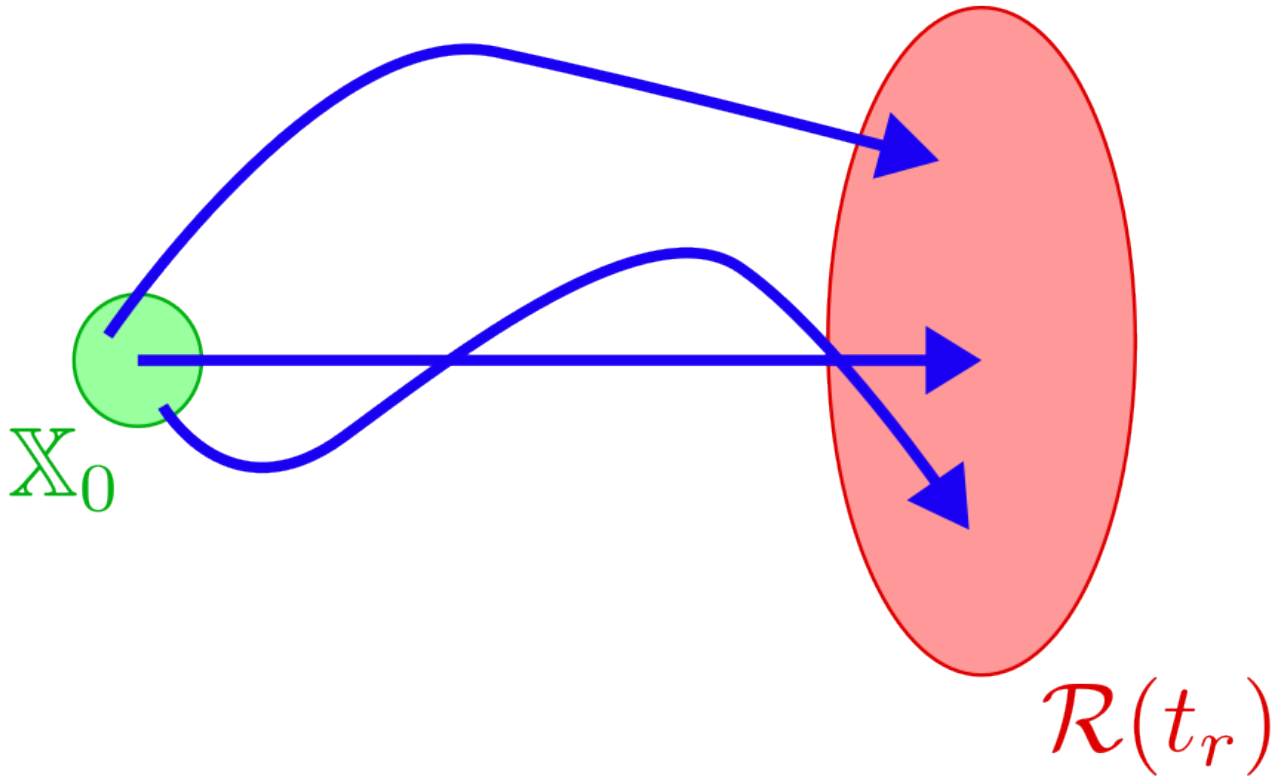
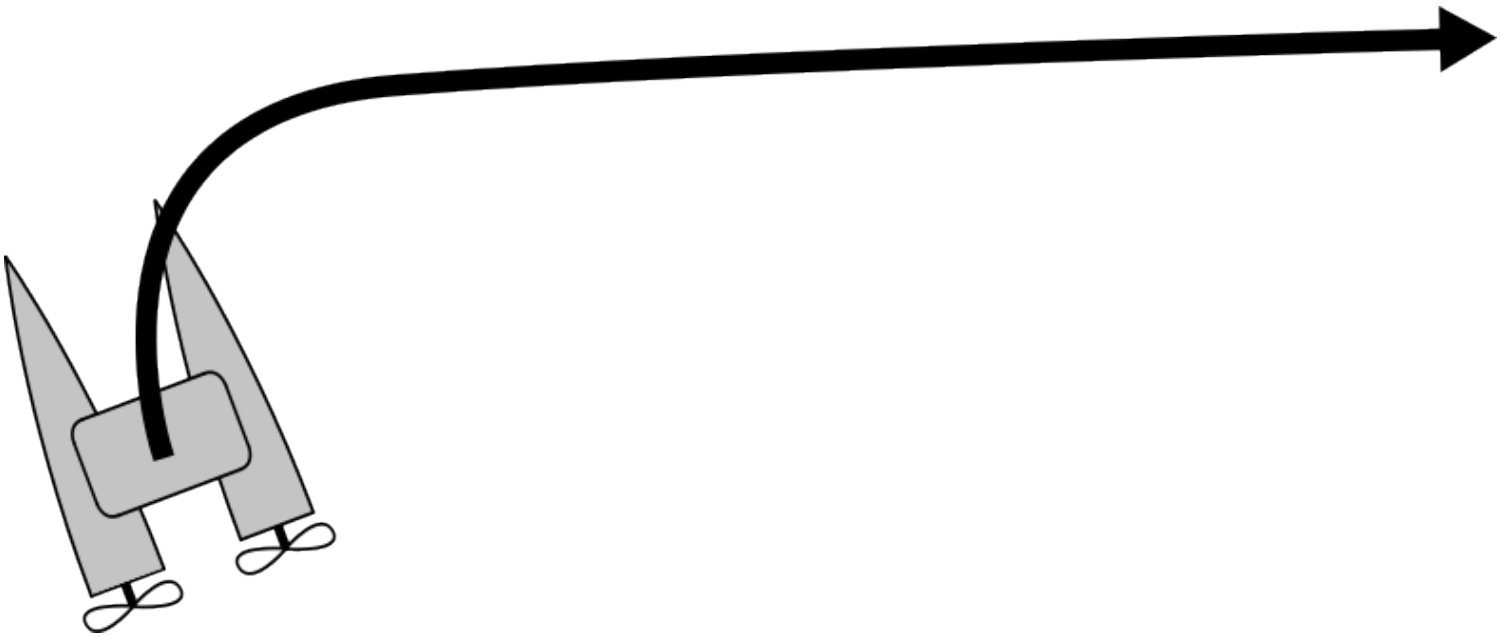
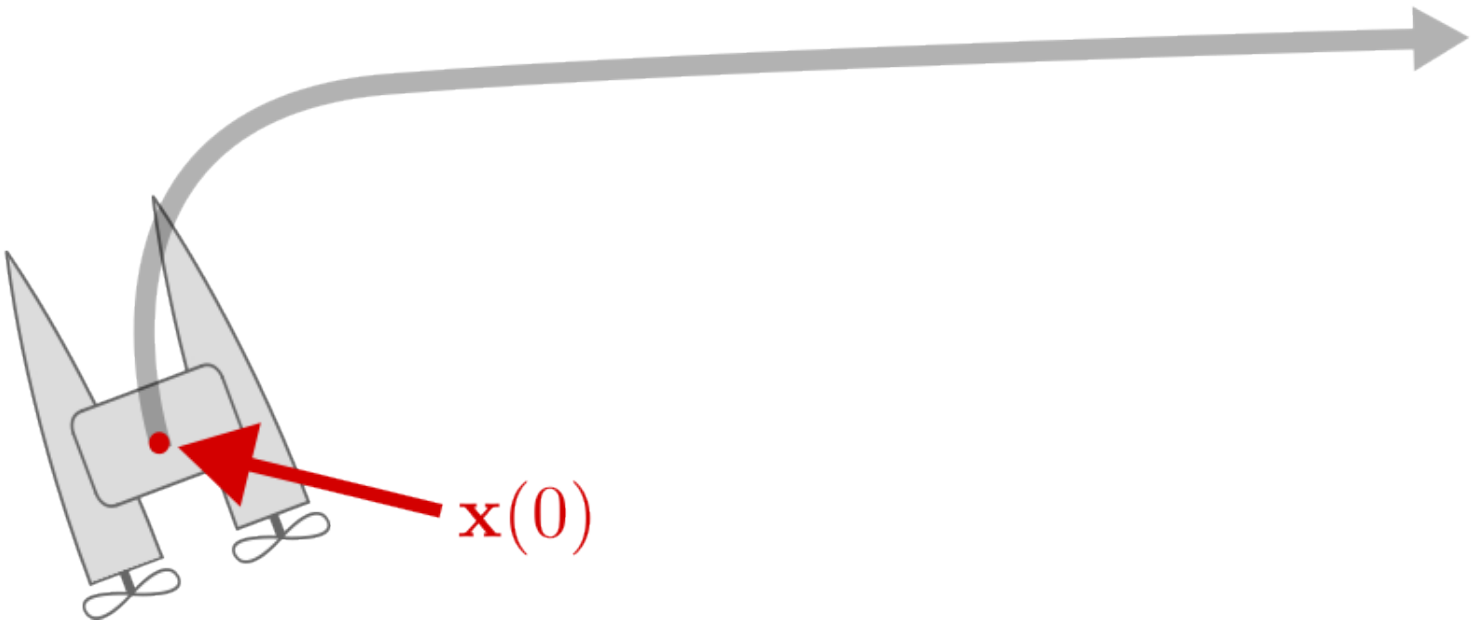

$\mathbb{X}_0$

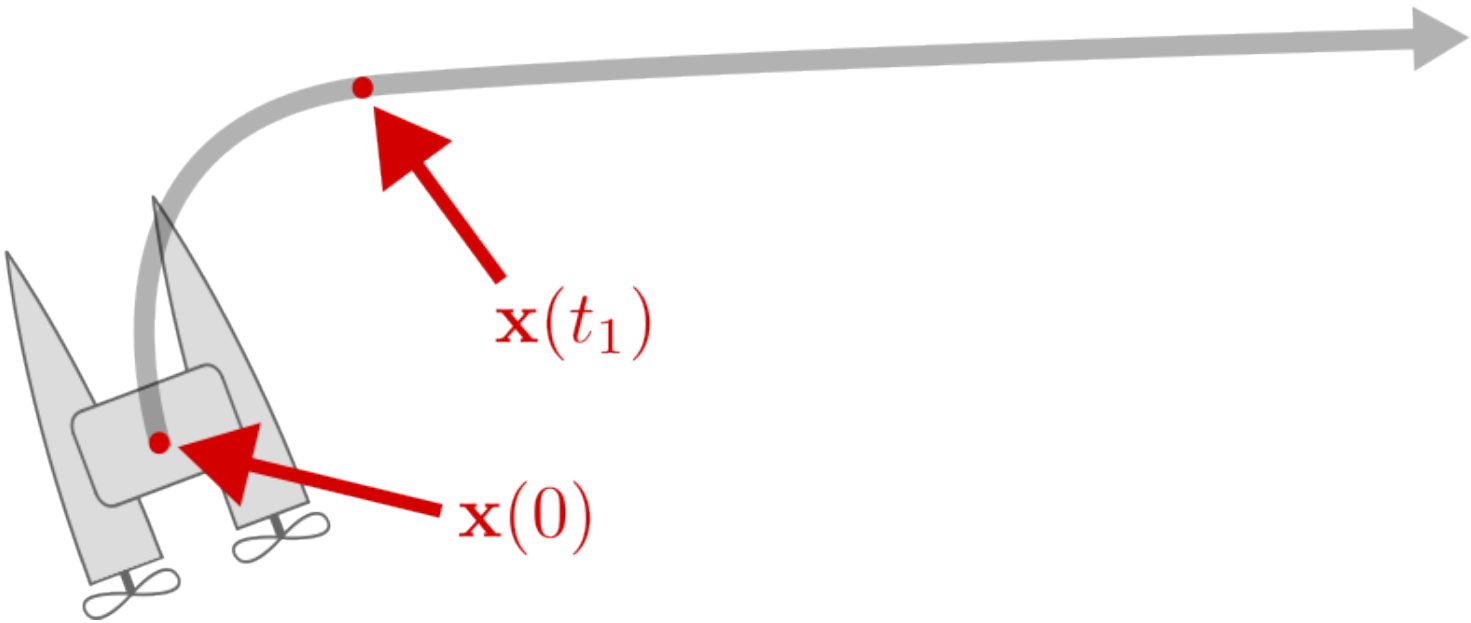

$\mathbb{X}_0$

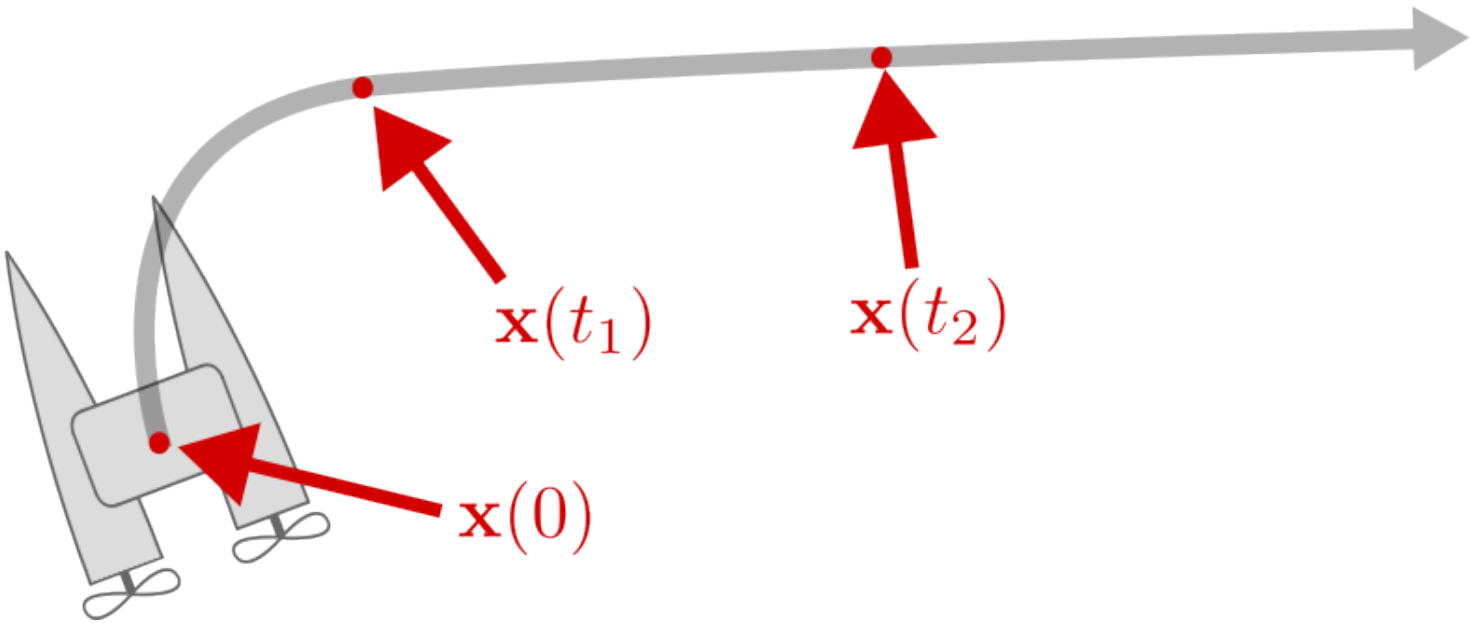

$\mathbb{X}_0$
$\mathcal{R}(t_r)$

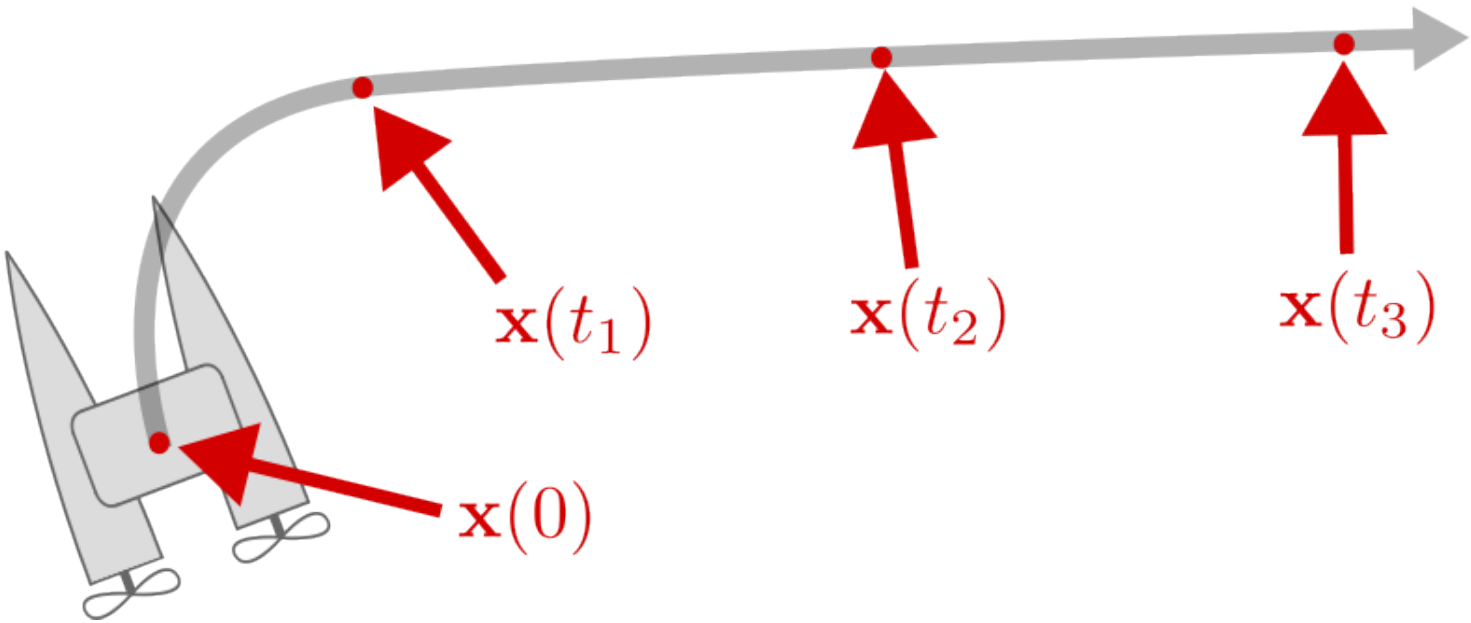
Representations



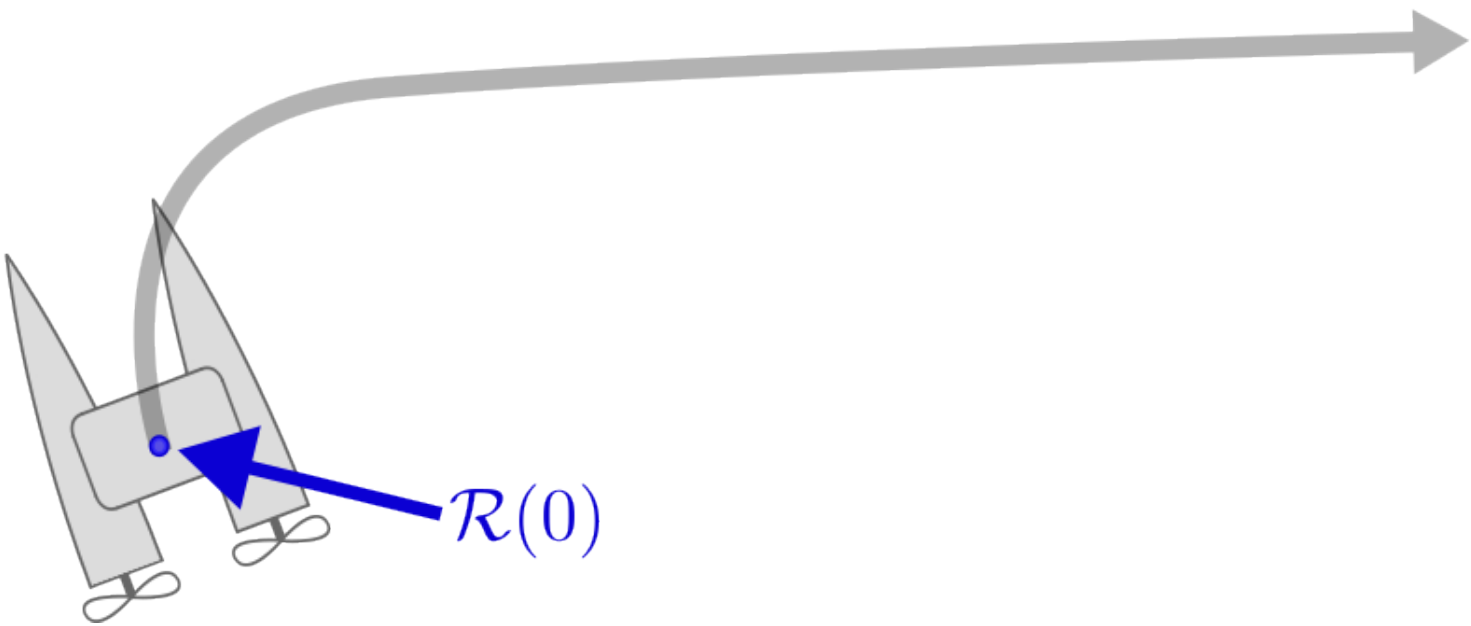

$\mathbf{x}(0)$

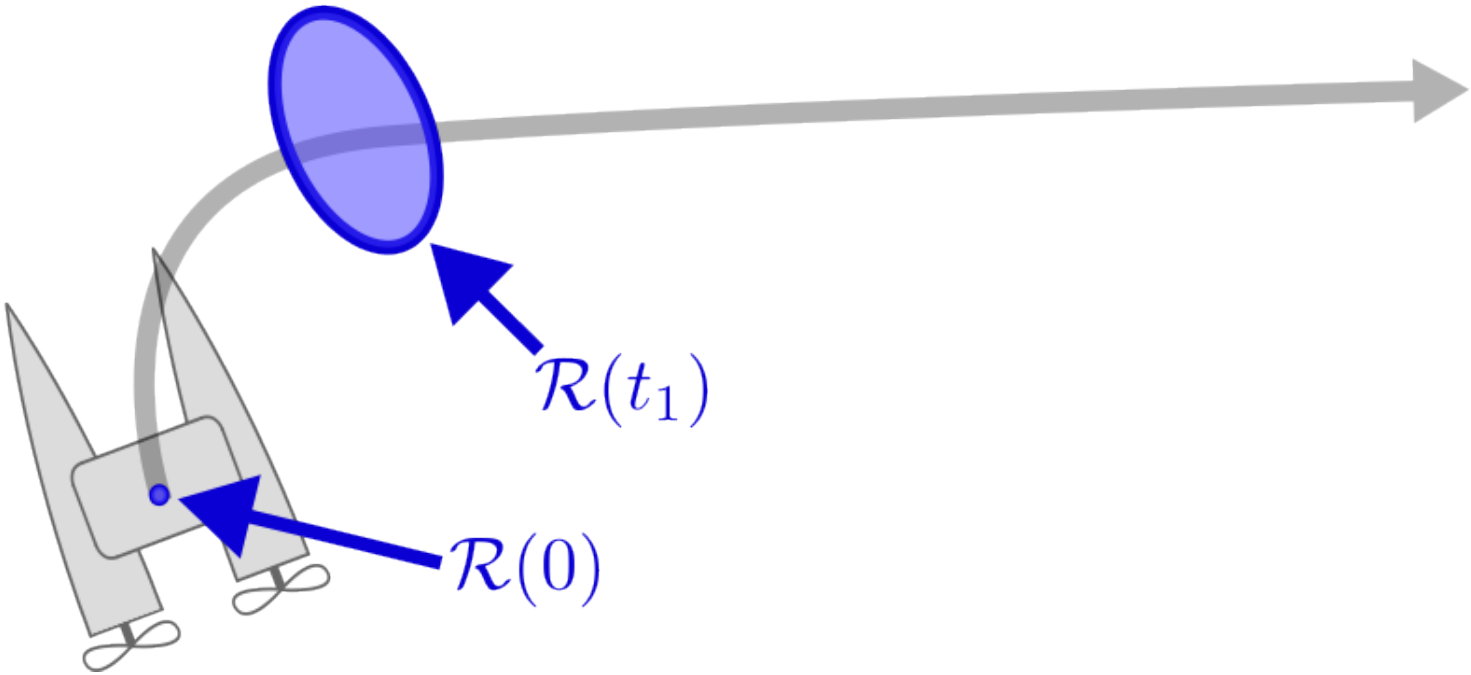

$\mathbf{x}(t_1)$
$\mathbf{x}(0)$

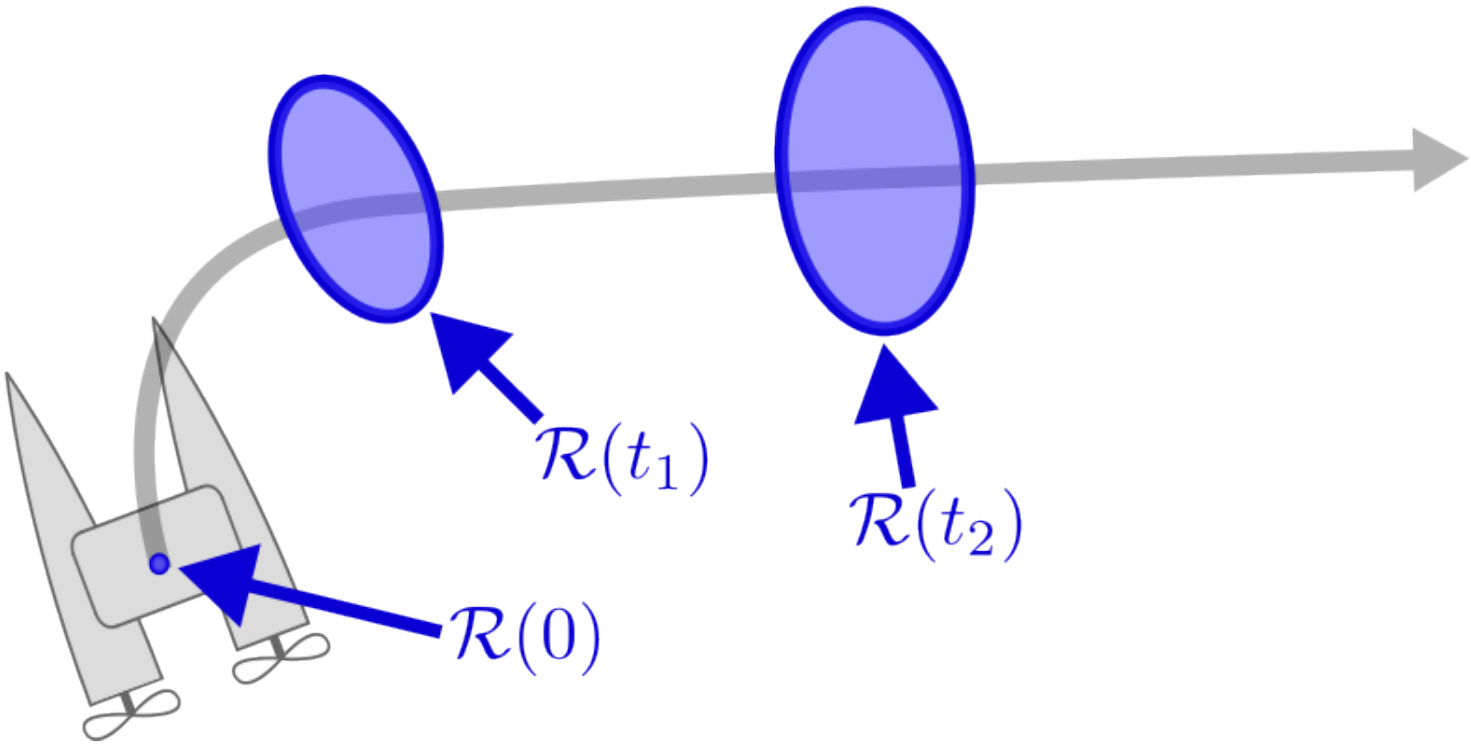

$\mathbf{x}(t_1)$
$\mathbf{x}(t_2)$
$\mathbf{x}(0)$

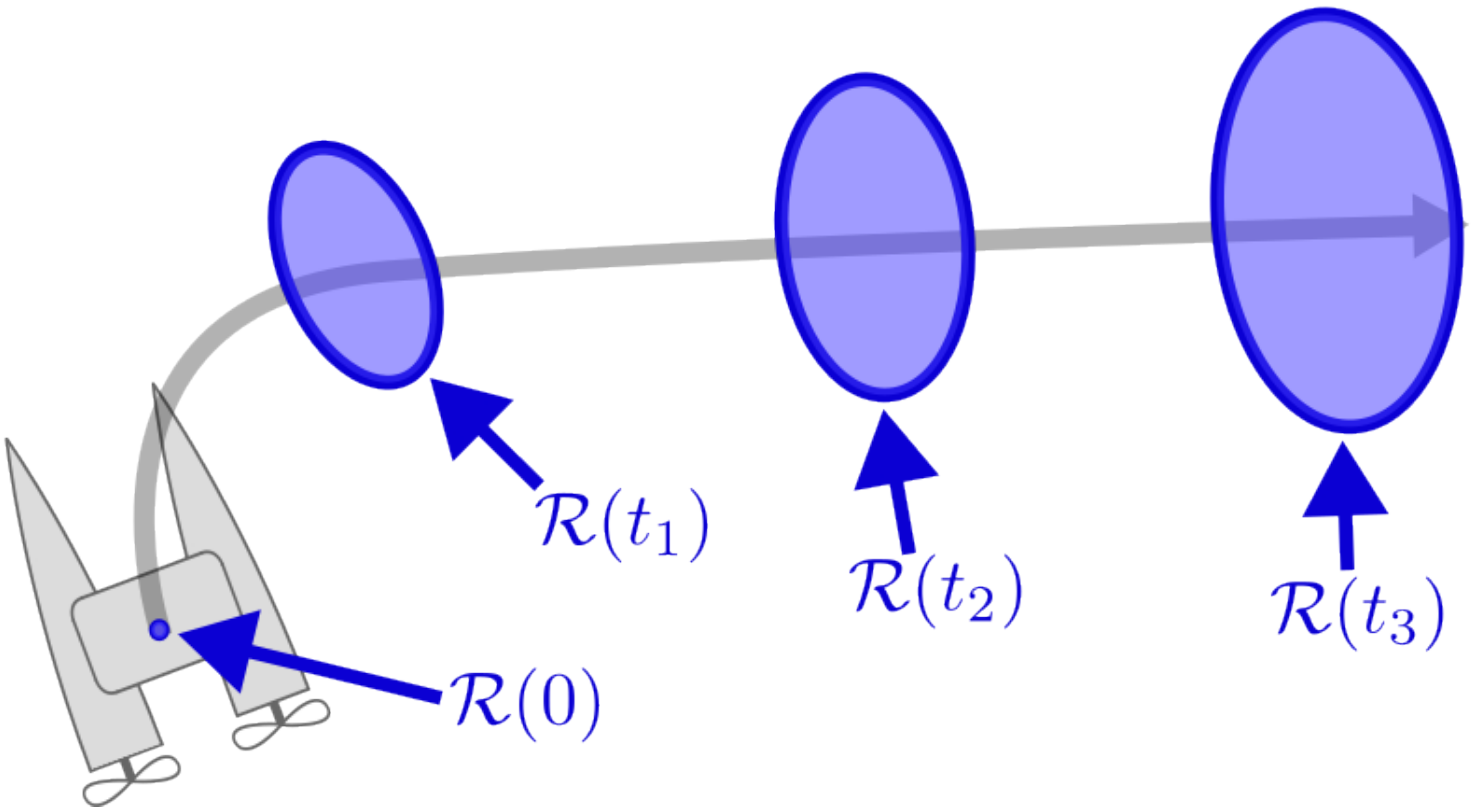

$\mathbf{x}(t_1)$
$\mathbf{x}(t_2)$
$\mathbf{x}(t_3)$
$\mathbf{x}(0)$

Discrete representation




$\mathcal{R}(t_1)$
$\mathcal{R}(0)$

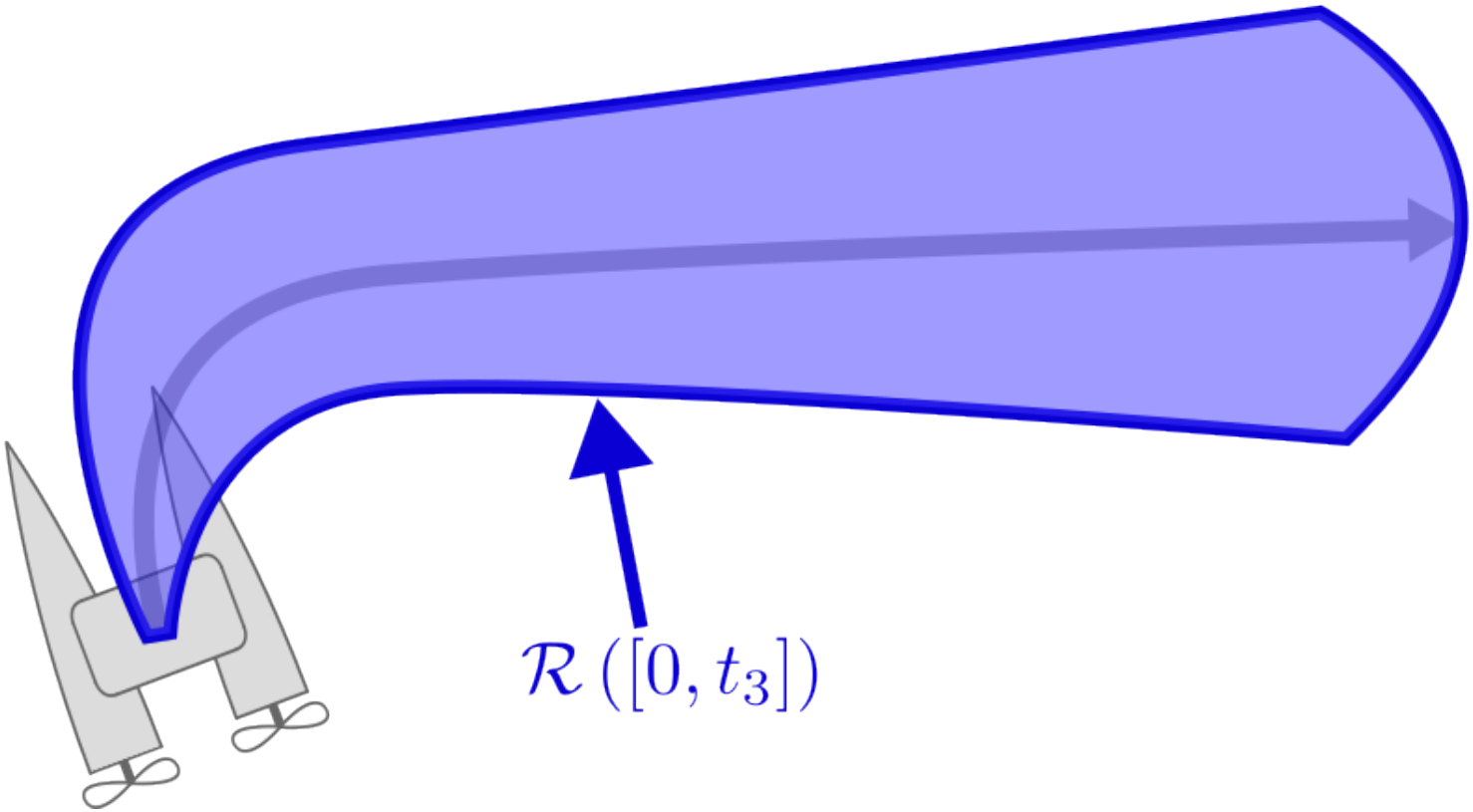

$\mathcal{R}(t_1)$
$\mathcal{R}(t_2)$
$\mathcal{R}(0)$


$\mathcal{R}(t_1)$
$\mathcal{R}(t_2)$
$\mathcal{R}(t_3)$
$\mathcal{R}(0)$

Can be obtained by a guaranteed intregration of the State Equation:

- Lohner Algorithm (see [1])
- CAPD library

Continuous representation



Can be obtained by using:

- Interval Analysis (codac library, see [2])

Finite time : Reaching an area

Figure: Base Situation

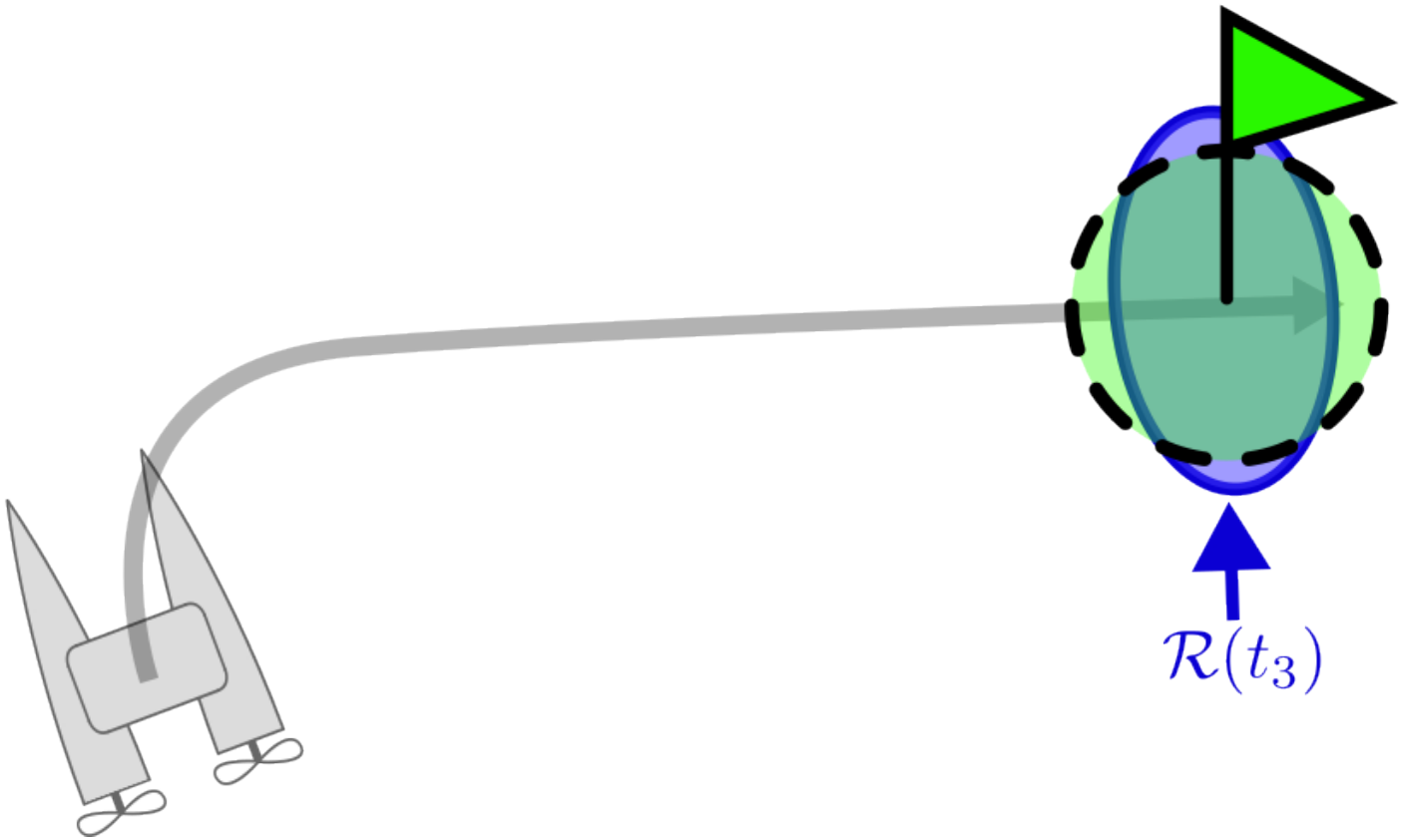


Figure: Unsafe Case

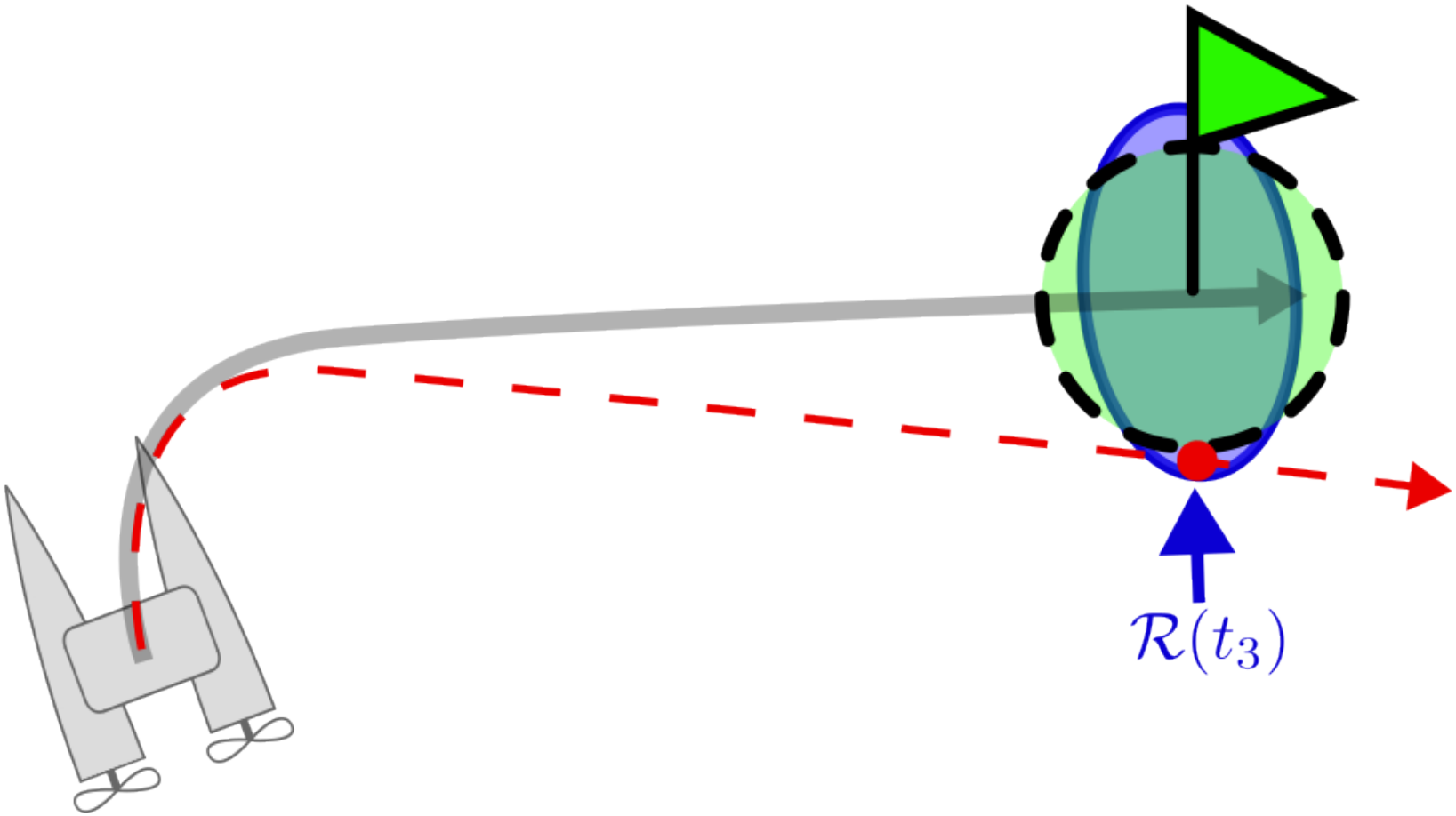


Figure: Unsafe Trajectory

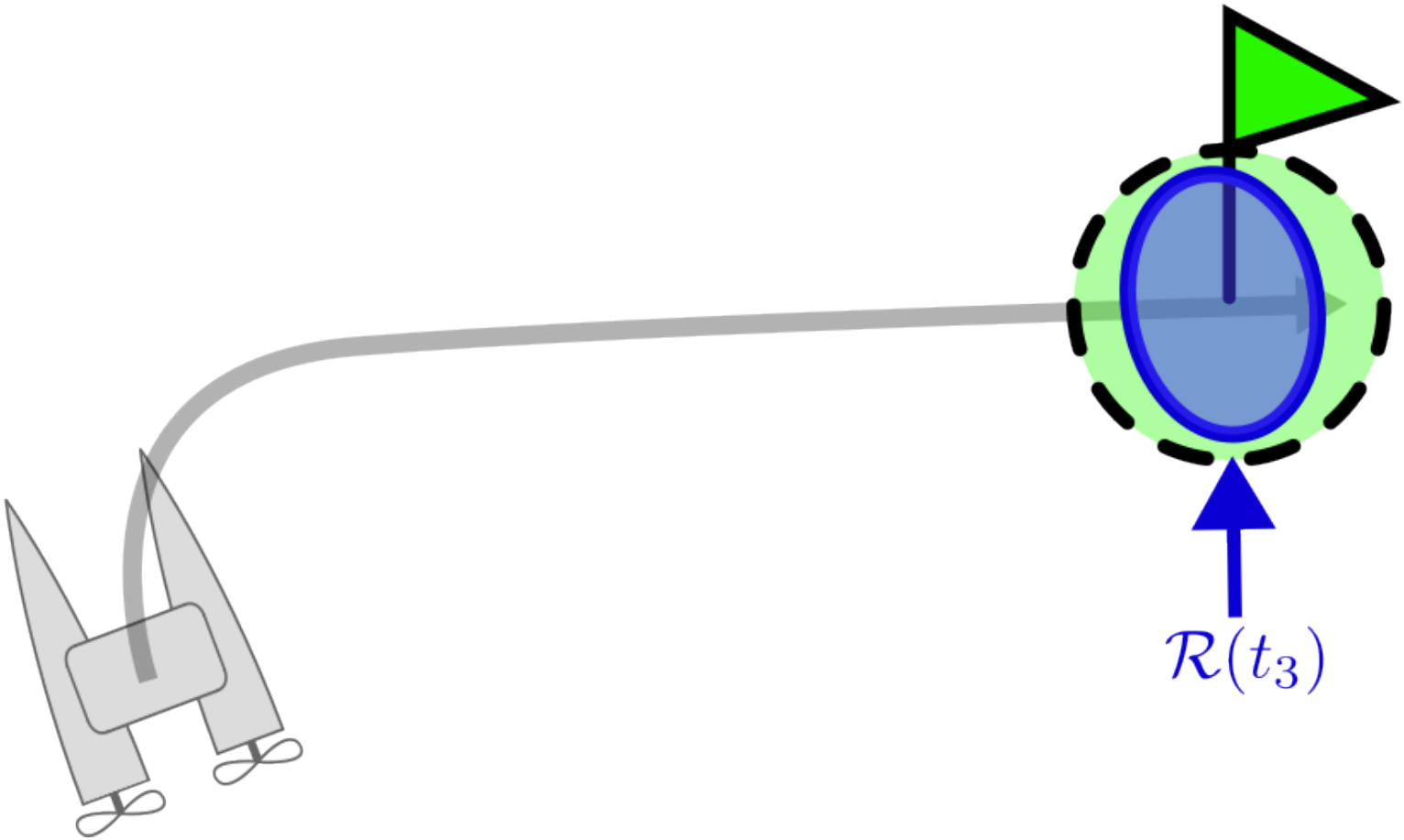


Figure: Safe Case

Infinite time : Obstacle avoidance

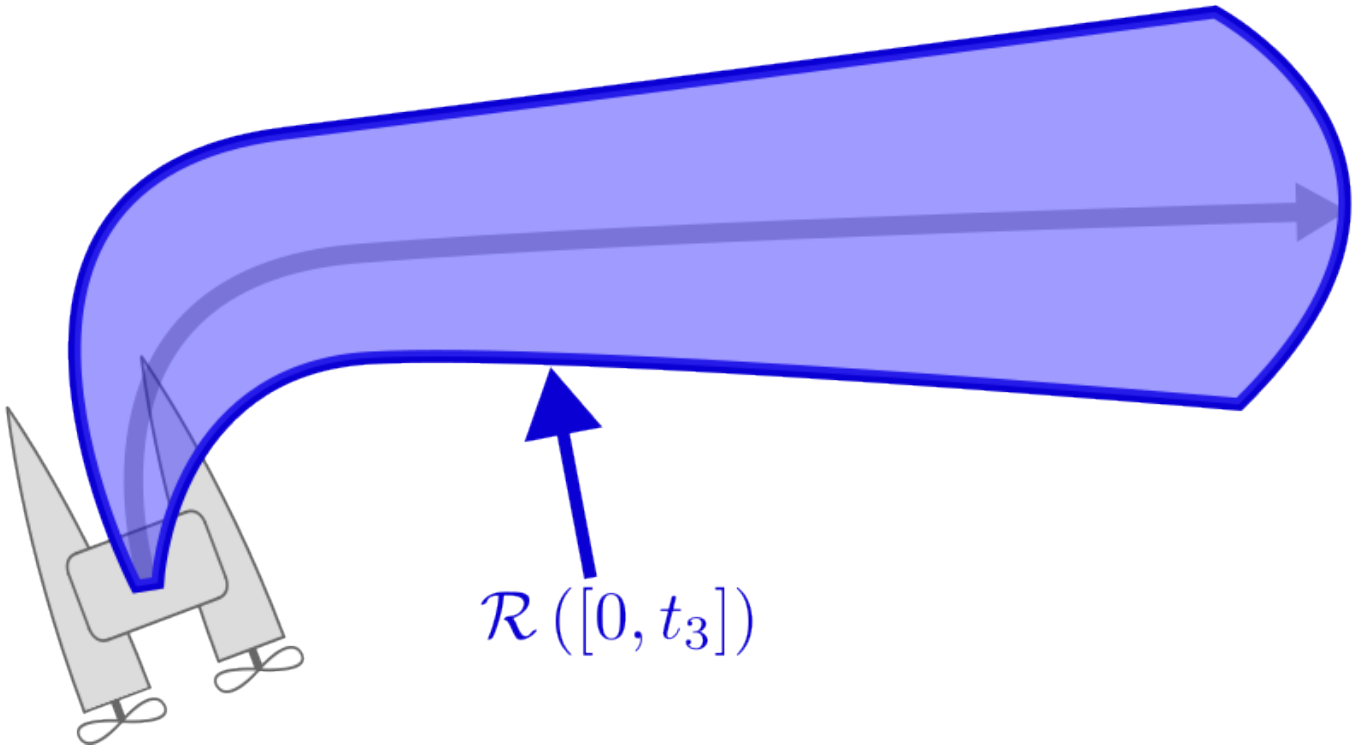
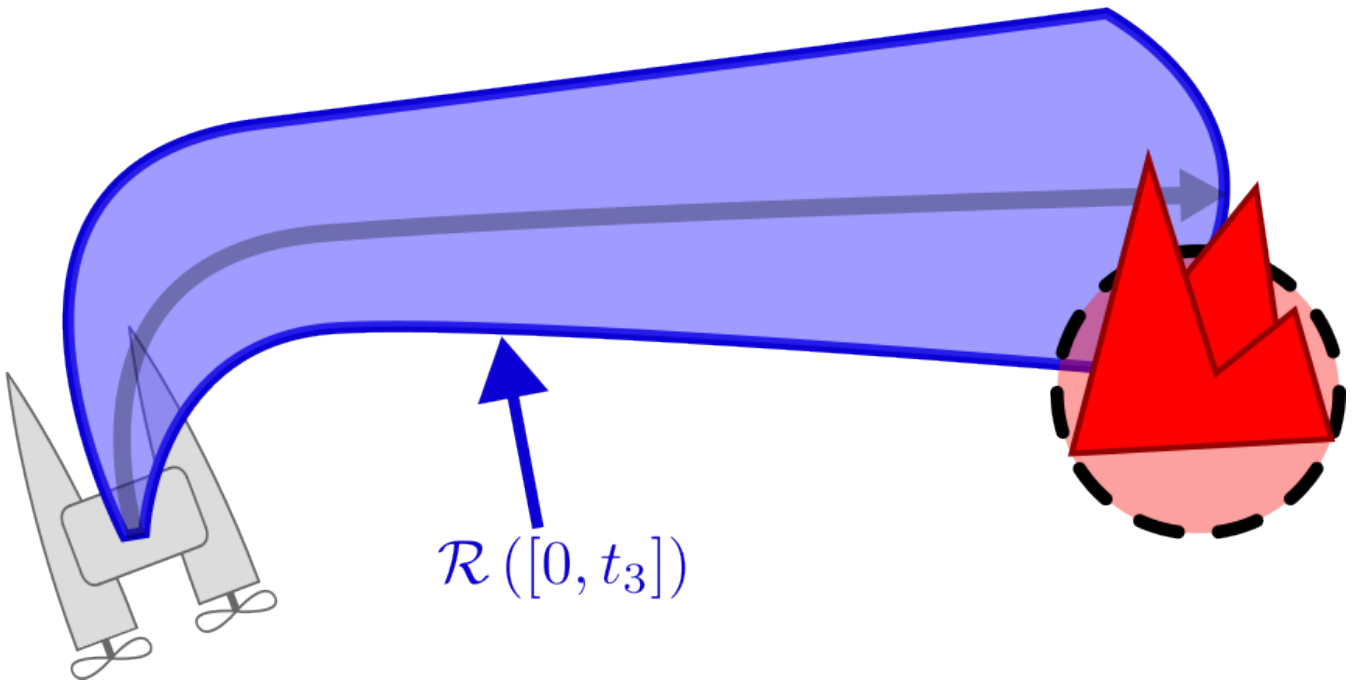


Figure: Continuous Representation



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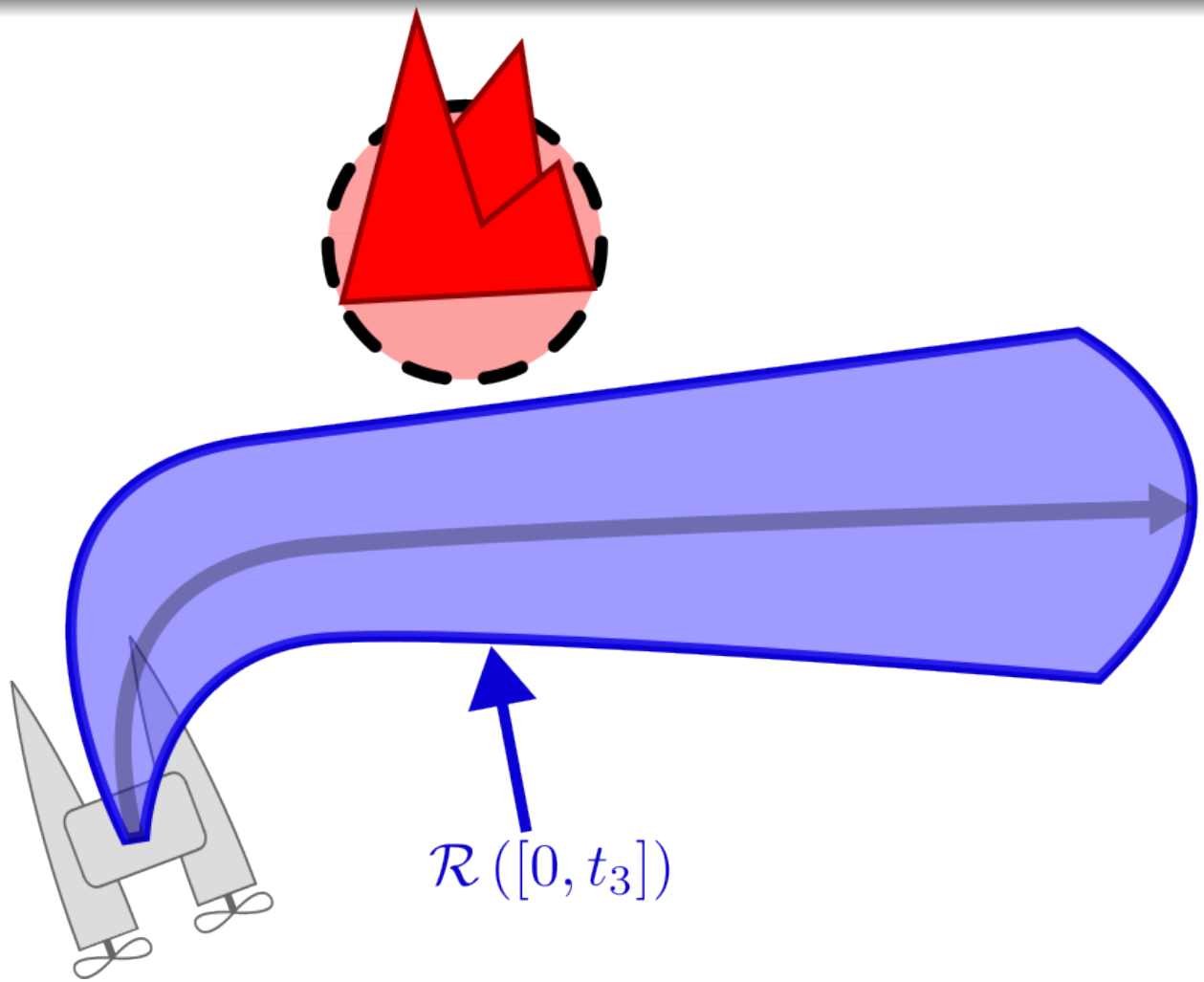

Figure: Safe Case

Conclusion

- Ensuring robot safety
- Dicrete / Continuous representation
- Different ways to estimate the Reachable Set (see References)

In my case Interval Analysis and optimal control

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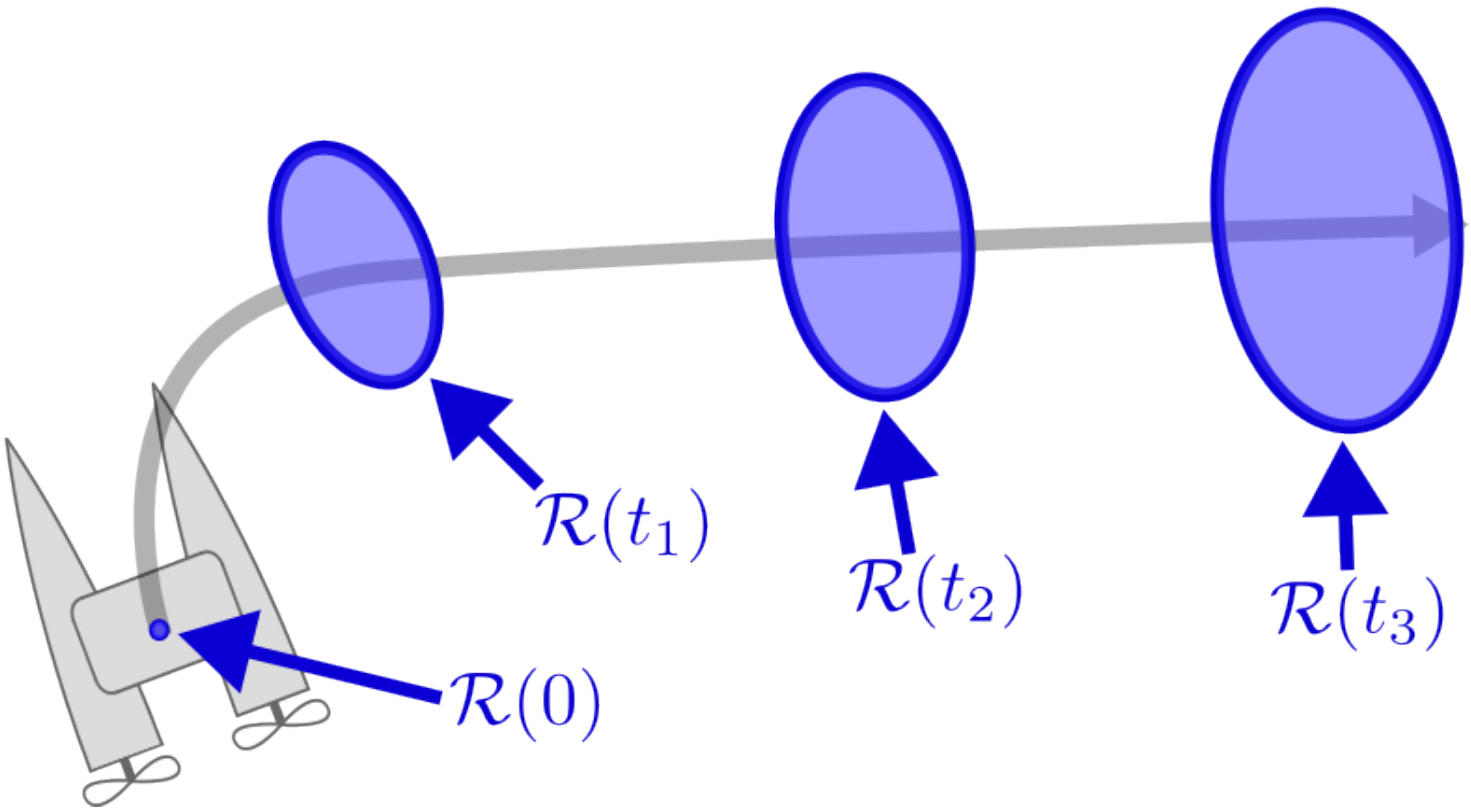
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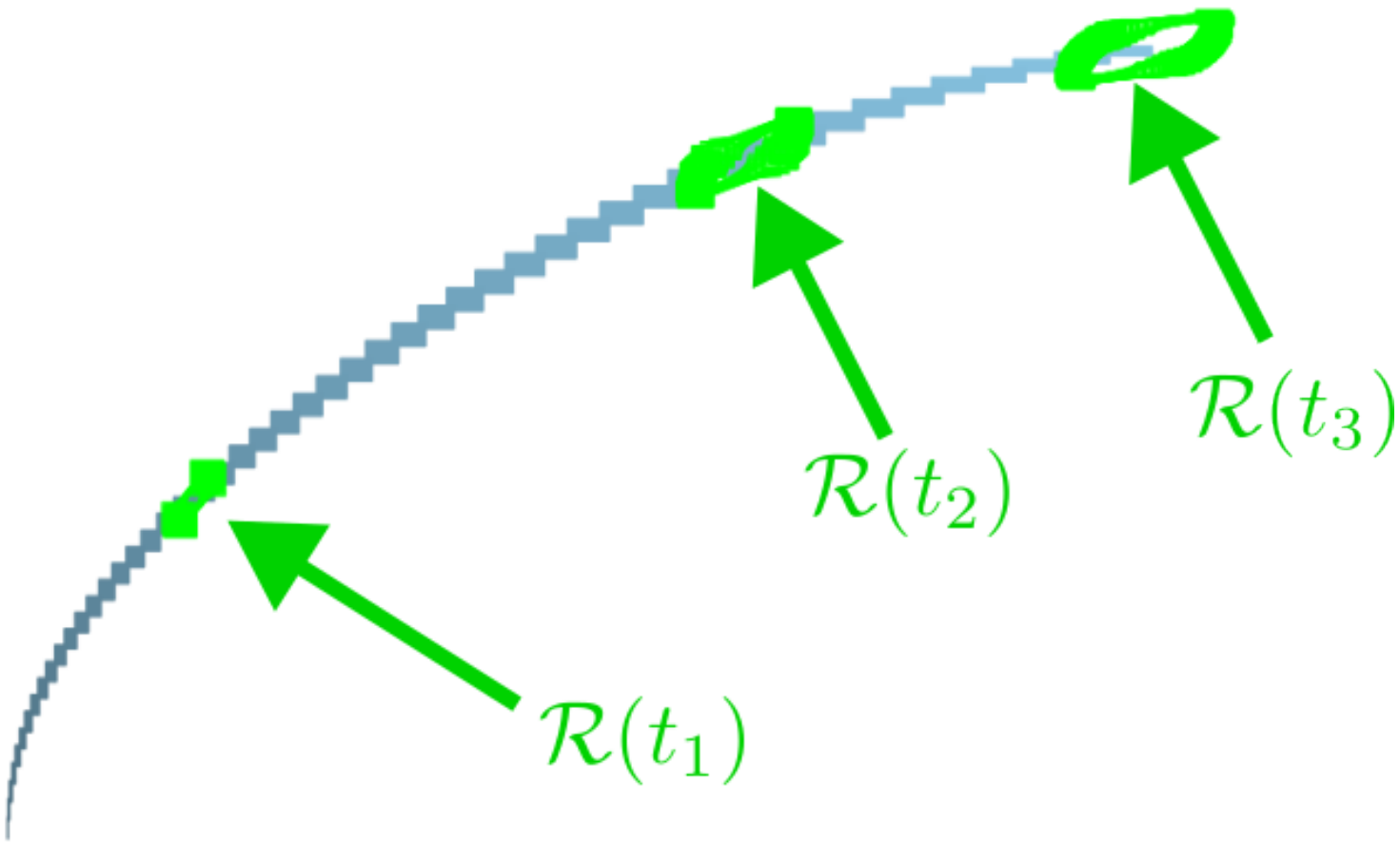
$\mathcal{R}(t_3)$
$\mathcal{R}(t_2)$
$\mathcal{R}(t_1)$

Thank you for listening

References

[1] Lohner R. J., Enclosing the solutions of ordinary initial and boundary value problems, *Computerarithmetic*, pp. 225–286, 1987.

[2] Rohou S., Jaulin L., Mihaylova L., Le Bars F, Veres S., Guaranteed computation of robot trajectories, *Robotics and Autonomous Systems*, Volume 93, 2017, Pages 76-84.

Lew T., Bonalli R., Pavone M., Exact Characterization of the Convex Hulls of Reachable Sets, *62nd IEEE Conference on Decision and Control (CDC 2023),* Dec 2023, Singapour, Singapore.

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