--
Matthias Althoff, Associate Professor
Cyber-Physical Systems Group
Department of Computer Science
Technische Universität München
phone: +49-89-289-18134
www: https://www.ce.cit.tum.de/cps/members/prof-dr-ing-matthias-althoff/

Here's my review:

Topic

Safe navigation in landmarked environments requires accounting for uncertainties in the dynamics of the robot, environmental disturbances, and errors in localization. This thesis focuses on autonomous vessels. While landmarks such as buoys, shore structures, or navigational beacons provide valuable information for positioning, the exact future trajectory of an autonomous vessel remains uncertain due to wind, waves, currents, actuator inaccuracies, and sensor noise. Reachability analysis is particularly relevant in this context because it computes the complete set of states that the vessel can possibly occupy over a future time horizon rather than only predicting a single trajectory. As a consequence, one can formally determine whether all admissible evolutions of the system remain within safe operating regions and avoid collisions with obstacles, shoreline boundaries, or restricted areas, even in the presence of uncertainty.

A further advantage of reachability analysis is that it naturally combines safety verification with decision making. Landmark measurements can be used to reduce uncertainty in the vessel state estimate, which in turn leads to smaller reachable sets and less conservative navigation strategies. At the same time, reachable sets provide explicit guarantees that planned maneuvers satisfy safety requirements before they are executed. This capability is particularly important for autonomous vessels operating in congested waterways, where interactions with other traffic participants and changing environmental conditions must be considered continuously. By reasoning about all possible future behaviors of the system instead of a nominal prediction, reachability analysis enables safety guarantees that are difficult to obtain with conventional planning methods alone.

Content

Chapter 1: The first chapter motivates the problem of safe navigation in environments where reliable global positioning is unavailable, with a particular emphasis on underwater robotics. It is argued that robots operating from landmark to landmark must account for uncertainty arising from disturbances, imperfect sensing, and localization errors. The chapter introduces reachable sets as a means to represent all possible robot states under bounded uncertainties and formulates the central objective of the thesis: guaranteeing that a robot can safely reach regions where localization becomes possible. Furthermore, the chapter establishes the connection between reachability analysis and the computation of the image of a set by a function, thereby

providing the conceptual framework for all subsequent contributions.

Chapter 2: This chapter presents the foundations of interval and set-based methods used throughout the thesis. After introducing interval analysis, inclusion functions, contractors, and paving algorithms, it discusses the sources of conservatism that arise when uncertainties are propagated through nonlinear systems. The chapter then reviews alternative set representations, with particular attention to zonotopes and parallelepipeds, highlighting the trade-off between computational efficiency and approximation quality. These notions form the computational backbone of the proposed methods, as they enable guaranteed enclosures while maintaining a manageable computational complexity.

Chapter 3: The third chapter introduces the topological and geometric tools required to reason about the boundary of sets. Concepts such as closures, interiors, compactness, manifolds, atlases, tangent spaces, normal spaces, and the Gauss map are reviewed with a focus on their practical relevance to reachability analysis. The key observation is that boundaries can often be studied in lower-dimensional spaces, reducing computational burden while preserving essential information. This perspective is later exploited to characterize reachable sets through their boundaries rather than through their full volume.

Chapter 4: This chapter formalizes the notion of reachable sets for dynamical systems subject to uncertainties and disturbances. After introducing differential equations, differential inclusions, flow functions, and finite-time reachability analysis, it reviews existing approaches to reachable set computation. Particular attention is given to boundary-based methods and to optimal-control formulations that characterize extremal trajectories. The chapter ultimately motivates the use of boundary propagation as a way to obtain less conservative approximations while keeping computational requirements under control.

Chapter 5: The first major contribution of the thesis is presented in this chapter. A novel parallelepiped inclusion function is derived for approximating the image of a set under a nonlinear mapping. By combining local linearization with a rigorously bounded inflation procedure, the method provides guaranteed outer enclosures with favorable convergence properties. The chapter further shows how images of set boundaries can be covered using gnomonic atlases and collections of small parallelepipeds, allowing both inner and outer approximations to be reconstructed. Finally, the approach is extended to ordinary differential equations, thereby establishing the connection to reachability analysis.

Chapter 6:Building upon the previous chapter, Chapter 6 develops a new boundary-based method for reachable set computation. The central result is an augmented differential equation whose solution generates the boundary of the reachable set directly. Under suitable smoothness assumptions, the exact reachable-set boundary is recovered; otherwise, a guaranteed outer approximation is obtained. The resulting boundary representation makes it possible to reconstruct both inner and outer approximations of the reachable set. The method is demonstrated on several examples, including neural dynamical systems and an autonomous surface vehicle, illustrating its ability to handle realistic robotic scenarios.

Chapter 7: This chapter addresses the computational burden associated with high-resolution boundary approximations. An adaptive strategy is proposed in which refinements are performed only in regions that are relevant for validating safety constraints. The resulting algorithms, Ad-PEIB and Ad-PEIBOS, combine parallelepipedic enclosures with contractors to concentrate computational resources where they are most needed. Applications to robotic manipulators, area-reaching problems, and workspace validation demonstrate that substantial computational savings can be achieved without compromising guarantees. The chapter also shows how unsafe regions can be identified and approximated when constraints cannot be validated globally.

Chapter 8: The eighth chapter investigates a limitation of boundary-based methods: the appearance of fake boundaries resulting from self-intersections of the image of a boundary. To address this issue, the notion of Box Chains is introduced. These structures decompose the domain into subsets over which the mapping remains injective, making self-intersection detection tractable. By combining topological reasoning with local injectivity tests, fake boundaries can be identified and removed automatically. As a result, significantly tighter inner approximations of image and reachable sets can be obtained, particularly in higher-dimensional settings where conservative over-approximations become problematic.

Chapter 9: This chapter discusses the practical realization of the proposed methods and emphasizes the importance of preserving mathematical guarantees at the numerical level. Issues related to floating-point arithmetic, rounding strategies, and validated computations are reviewed. The implementation of the developed algorithms within the CODAC library is described, together with supporting software tools such as CAPD and CGAL. The chapter also documents the publicly available code used to generate the numerical examples and figures of the thesis, ensuring reproducibility of the presented results.

Chapter 10: The final chapter summarizes the contributions of the thesis and discusses their implications for the validation of robotic missions. The proposed parallelepiped-based framework enables the computation of both inner and outer approximations of image and reachable sets while maintaining formal guarantees. Additional developments, including adaptive refinements and fake-boundary removal, improve both efficiency and accuracy. The chapter concludes by outlining several directions for future research, including tighter integrations of the developed techniques, broader applicability to nonsmooth problems, and systematic comparisons with existing reachability-analysis tools.

Evaluation

A major strength of the thesis is that it proposes several original algorithms for the characterization of image and reachable sets using boundary-based representations and parallelepipedic enclosures. Nevertheless, one of the central challenges in reachability analysis is scalability with respect to the system dimension. The computational burden of most existing approaches grows rapidly when the state dimension increases, which is often the limiting factor for practical applications. For this reason, it would have been highly beneficial to derive the computational complexity of the proposed methods with respect to the system dimension and to discuss how the algorithms compare theoretically to existing reachability techniques. While

Chapter 7 contains a complexity discussion for some adaptive procedures, the analysis remains largely independent of the state dimension of the underlying dynamical system. In addition, the experimental results are restricted to relatively low-dimensional examples, making it difficult to assess whether the proposed methods remain competitive for larger-scale systems.

Another aspect that weakens the evaluation is the lack of comparisons with established reachability-analysis tools. Over the last years, frameworks such as Ariadne, CORA, DynIbex, and JuliaReach have become widely used benchmarks for assessing new reachability techniques. A direct comparison with these tools would have helped to position the proposed methods with respect to the current state of the art and would have provided valuable insight into their practical advantages and limitations. Similarly, it would have been beneficial to evaluate the approach on standard benchmark problems, such as those collected within ARCH-COMP.

Finally, the novelty of the individual contributions is not always communicated clearly enough. While the thesis contains a number of interesting ideas and technically sophisticated developments, several chapters do not explicitly discuss how the presented results extend existing work and which limitations of previous approaches are overcome. More systematic comparisons to related methods and a clearer discussion of the scientific advances would be appreciated. In addition, the manuscript contains a noticeable number of spelling mistakes, grammatical inaccuracies, and stylistic inconsistencies.
---

favorable opinion

Matthias Althoff